EXAMPLE 3.1

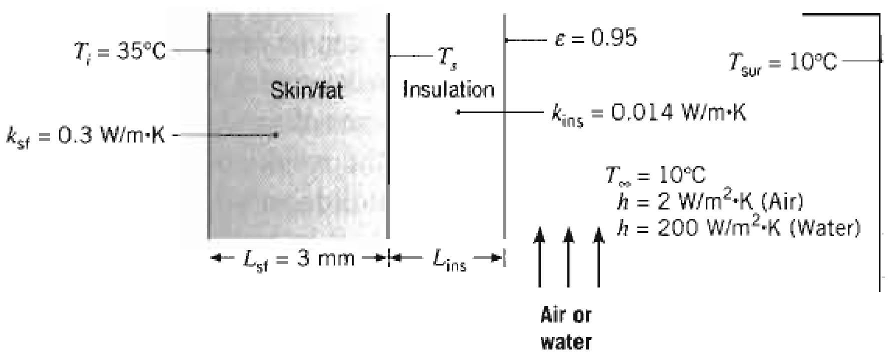
To reduce the heat loss rate, the person wears special sporting gear (snow suit and wet suit) made from a nanostructured silica aerogel insulation with an extremely low thermal conductivity of 0.014 W/m · K. The emissivity of the outer surface of the snow and wet suits is 0.95. What thickness of aerogel insulation is needed to reduce the heat loss rate to 100 W (a typical metabolic heat generation rate) in air and water? What are the resulting skin temperatures?

SOLUTION

Known: Inner surface temperature of a skin/fat layer of known thickness, thermal conductivity, and surface area. Thermal conductivity and emissivity of snow and wet suits. Ambient conditions.

Find: Insulation thickness needed to reduce heat loss rate to 100 W and corresponding skin temperature.

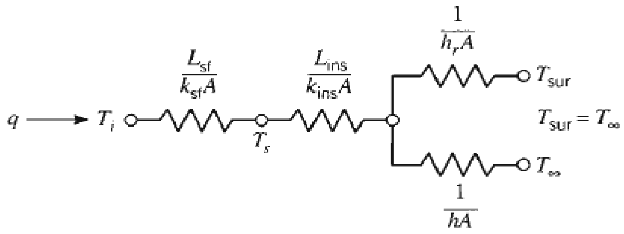
Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional heat transfer by conduction through the skin/fat and insulation layers.
3. Contact resistance is negligible.
4. Thermal conductivities are uniform.
5. Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
6. Liquid water is opaque to thermal radiation.
7. Solar radiation is negligible.

Analysis: The thermal circuit can be constructed by recognizing that resistance to heat flow is associated with conduction through the skin/fat and insulation layers and convection and radiation at the outer surface. Accordingly, the circuit and the resistances are of the following form (with $h_r = 0$ for water):



The total thermal resistance needed to achieve the desired heat loss rate is found from Equation 3.19,

$$R_{tot} = \frac{T_i - T_\infty}{q} = \frac{(35-10)\ \text{K}}{100\ \text{W}} = 0.25\ \text{K/W}$$

The total thermal resistance between the inside of the skin/fat layer and the cold surroundings includes conduction resistances for the skin/fat and insulation layers and an effective resistance associated with convection and radiation, which act in parallel. Hence,

$$R_{tot} = \frac{L_{sf}}{k_{sf}A} + \frac{L_{ins}}{k_{ins}A} + \left(\frac{1}{1/hA} + \frac{1}{1/h_rA}\right)^{-1} = \frac{1}{A}\left(\frac{L_{sf}}{k_{sf}} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h+h_r}\right)$$

This equation can be solved for the insulation thickness.

Air

The radiation heat transfer coefficient is approximated as having the same value as in Example 1.6: $h_r = 5.9\ \text{W/m}^2\cdot\text{K}$.

$$L_{ins} = k_{ins}\left[AR_{tot} - \frac{L_{sf}}{k_{sf}} - \frac{1}{h+h_r}\right]$$

$$= 0.014\ \text{W/m}\cdot\text{K}\left[1.8\ \text{m}^2 \times 0.25\ \text{K/W} - \frac{3\times10^{-3}\text{m}}{0.3\ \text{W/m}\cdot\text{K}} - \frac{1}{(2+5.9)\ \text{W/m}^2\cdot\text{K}}\right]$$

$$= 0.0044\ \text{m} = 4.4\ \text{mm}$$ ◁

Water

$$L_{ins} = k_{ins}\left[AR_{tot} - \frac{L_{sf}}{k_{sf}} - \frac{1}{h}\right]$$

$$= 0.014\ \text{W/m}\cdot\text{K}\left[1.8\ \text{m}^2 \times 0.25\ \text{K/W} - \frac{3\times10^{-3}\ \text{m}}{0.3\ \text{W/m}\cdot\text{K}} - \frac{1}{200\ \text{W/m}^2\cdot\text{K}}\right]$$

$$= 0.0061\ \text{m} = 6.1\ \text{mm}$$ ◁

These required thicknesses of insulation material can easily be incorporated into the snow and wet suits.

The skin temperature can be calculated by considering conduction through the skin/fat layer:

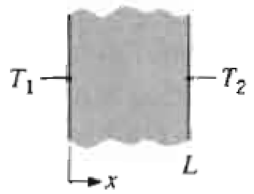
$$q = \frac{k_{sf} A (T_i - T_s)}{L_{sf}}$$

or solving for T_s,

$$T_s = T_i - \frac{q L_{sf}}{k_{sf} A} = 35°\text{C} - \frac{100 \text{ W} \times 3 \times 10^{-3} \text{ m}}{0.3 \text{ W/m} \cdot \text{K} \times 1.8 \text{ m}^2} = 34.4°\text{C} \quad \triangleleft$$

The skin temperature is the same in both cases because the heat loss rate and skin/fat properties are the same.

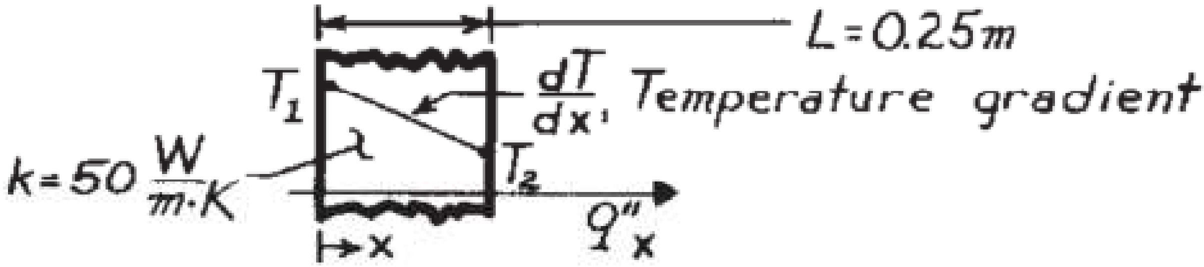
EXAMPLE 2. Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity $k = 50$ W/m · K and a thickness $L = 0.25$ m, with no internal heat generation.



Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.

Case	T_1(°C)	T_2(°C)	dT/dx (K/m)
1	50	−20	
2	−30	−10	
3	70		160
4		40	−80
5		30	200

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.
FIND: Unknowns for various temperature conditions and sketch distribution.



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

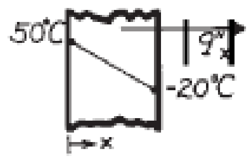
ANALYSIS: The rate equation and temperature gradient for this system are

$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a) $$\frac{dT}{dx} = \frac{(-20-50)\,K}{0.25m} = -280\ K/m$$

$$q''_x = -50\frac{W}{m\cdot K}\times\left[-280\frac{K}{m}\right] = 14.0\ kW/m^2.$$



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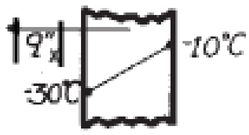
(b) $$\frac{dT}{dx} = \frac{(-10-(-30))\,K}{0.25m} = 80\ K/m$$

$$q''_x = -50\frac{W}{m\cdot K}\times\left[80\frac{K}{m}\right] = -4.0\ kW/m^2.$$



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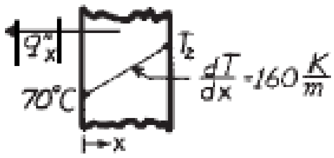
(c) $$q''_x = -50\frac{W}{m\cdot K}\times\left[160\frac{K}{m}\right] = -8.0\ kW/m^2$$

$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.25m\times\left[160\frac{K}{m}\right] + 70^\circ C.$$



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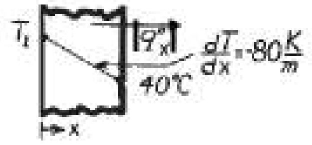
(d) $q''_x = -50 \frac{W}{m \cdot K} \times \left[-80 \frac{K}{m}\right] = 4.0 \text{ kW/m}^2$

$T_1 = T_2 - L \cdot \frac{dT}{dx} = 40^\circ C - 0.25m \left[-80 \frac{K}{m}\right]$

$T_1 = 60^\circ C.$



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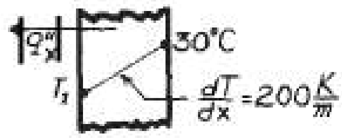
(e) $q''_x = -50 \frac{W}{m \cdot K} \times \left[200 \frac{K}{m}\right] = -10.0 \text{ kW/m}^2$

$T_1 = T_2 - L \cdot \frac{dT}{dx} = 30^\circ C - 0.25m \left[200 \frac{K}{m}\right] = -20^\circ C.$



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T2 = 110 C.

EXAMPLE 3.

3.2 The rear window of an automobile is defogged by passing warm air over its inner surface.

(a) If the warm air is at $T_{\infty,i} = 40°C$ and the corresponding convection coefficient is $h_i = 30 \text{ W/m}^2 \cdot K$, what are the inner and outer surface temperatures of 4-mm-thick window glass, if the outside ambient air temperature is $T_{\infty,o} = -10°C$ and the associated convection coefficient is $h_o = 65 \text{ W/m}^2 \cdot K$?

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

$$q'' = \frac{50^\circ C}{(0.0154 + 0.0029 + 0.0333)\,m^2 \cdot K/W} = 969\,W/m^2.$$

Hence, with $q'' = h_i\left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ C - \frac{969\,W/m^2}{30\,W/m^2 \cdot K} = 7.7^\circ C \qquad <$$

Similarly for the outer surface temperature with $q'' = h_o\left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} + \frac{q''}{h_o} = -10^\circ C + \frac{969\,W/m^2}{65\,W/m^2 \cdot K} = 4.9^\circ C \qquad <$$

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ C - \left(-10^\circ C\right)}{\frac{1}{65\,W/m^2 \cdot K} + \frac{0.004\,m}{1.4\,W/m \cdot K} + \frac{1}{30\,W/m^2 \cdot K}}$$

EXAMPLE 4.

A spherical, thin-walled metallic container is used to store liquid nitrogen at 77 K. The container has a diameter of 0.5 m and is covered with an evacuated, reflective insulation composed of silica powder. The insulation is 25 mm thick, and its outer surface is exposed to ambient air at 300 K. The convection coefficient is known to be 20 $W/m^2 \cdot K$. The latent heat of vaporization and the density of liquid nitrogen are 2×10^5 J/kg and 804 kg/m^3, respectively.

1. What is the rate of heat transfer to the liquid nitrogen?
2. What is the rate of liquid boil-off?

SOLUTION

Known: Liquid nitrogen is stored in a spherical container that is insulated and exposed to ambient air.

Find:

1. The rate of heat transfer to the nitrogen.

Assumptions:

1. Steady-state conditions.
2. One-dimensional transfer in the radial direction.
3. Negligible resistance to heat transfer through the container wall and from the container to the nitrogen.
4. Constant properties.
5. Negligible radiation exchange between outer surface of insulation and surroundings.

Properties: Table A.3, evacuated silica powder (300 K): $k = 0.0017$ W/m · K.

Analysis:

1. The thermal circuit involves a conduction and convection resistance in series and is of the form

where, from Equation 3.36,

$$R_{t,\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

and from Equation 3.9

$$R_{t,\text{conv}} = \frac{1}{h4\pi r_2^2}$$

The rate of heat transfer to the liquid nitrogen is then

$$q = \frac{T_{\infty,2} - T_{\infty,1}}{(1/4\pi k)[(1/r_1) - (1/r_2)] + (1/h4\pi r_2^2)}$$

Hence,

$$q = [(300 - 77)\text{ K}] \div \left[\frac{1}{4\pi(0.0017\text{ W/m}\cdot\text{K})}\left(\frac{1}{0.25\text{ m}} - \frac{1}{0.275\text{ m}}\right) + \frac{1}{(20\text{ W/m}^2\cdot\text{K})4\pi(0.275\text{ m})^2}\right]$$

$$q = \frac{223}{17.02 + 0.05}\text{ W} = 13.06\text{ W}$$ <

Example 5:

The total incident radiant energy upon a body which partiallly reflects, absorbs and transmits radiant energy is 2200 W/m^2. Of this amount, 450 W/m^2 is reflected and 900 W/m^2 is absorbed by the body. Find the transmissivity.

$$\tau = 1 - \rho - \alpha = 1 - \frac{450}{2200} - \frac{900}{2200} = 0.386$$

Example 6:

EXAMPLE 2.2: Determination of the insulation layer thickness of a house wall

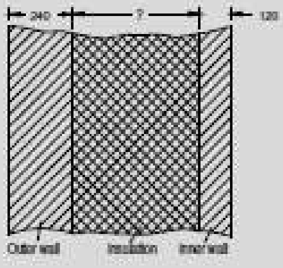
The wall of a house consists of an outer brick layer of 240 mm thickness and an inner layer of 120 mm thickness. Between the two walls there is mineral fibre insulation layer. The thermal conductivity of the inner and outer wall is 1 W/(m K), that of the insulation 0.035 W/(m K). The overall heat transfer coefficient of the multiple layer house wall shall not exceed 0.3 W/(m^2 K).

Find

The required insulation thickness.

Solution

Schematic See sketch.



Assumptions

- The thermal conductivities of all layers are areally homogeneous and independent from the temperature.
- No heat losses on sides of the wall.

Analysis

The overall heat transfer coefficient is given with Equation (2.15).

$$\frac{1}{k} = \sum_{i=1}^{n} \frac{s_i}{\lambda_i} = \frac{s_1}{\lambda_1} + \frac{s_2}{\lambda_2} + \frac{s_3}{\lambda_3}$$

In this example the overall heat transfer coefficient is known, the thickness of the insulation layer s_2 is to be determined. Therefore, with the above equation s_2 can be calculated.

$$s_2 = \left(\frac{1}{k} - \frac{s_1}{\lambda_1} - \frac{s_3}{\lambda_3}\right) \cdot \lambda_2 =$$

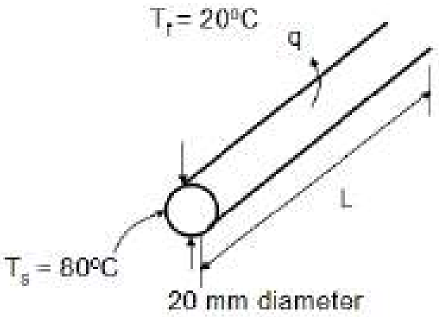
$$= \left(\frac{1}{0.3} - \frac{0.24}{1} - \frac{0.12}{1}\right) \cdot \frac{m^2 \cdot K}{W} \cdot 0.035 \cdot \frac{W}{m^2 \cdot K} = \mathbf{0.104\ m}$$

Discussion

The insulation layer is the main heat transfer resistance. Its heat transfer coefficient of 0.337 W/(m^2 · K) is only 12 % higher as the overall heat transfer coefficient.

Example 7:

A 20 mm diameter copper pipe is used to carry heated water, the external surface of the pipe is subjected to a convective heat transfer coefficient of $h = 6\ W/m^2K$, find the heat loss by convection per metre length of the pipe when the external surface temperature is 80°C and the surroundings are at 20°C. Assuming black body radiation what is the heat loss by radiation?



Solution

$$q_{conv} = h(T_s - T_f) = 6(80 - 20) = 360\ W/m^2$$

For 1 metre length of the pipe:

$$Q_{conv} = q_{conv}A = q_{conv} \times 2\pi r = 360 \times 2 \times \pi \times 0.01 = 22.6\ W/m$$

For radiation, assuming black body behaviour:

$$q_{rad} = \sigma(T_s^4 - T_f^4)$$

$$q_{rad} = 5.67 \times 10^{-8}(353^4 - 293^4)$$

$$q_{rad} = 462\ W/m^2$$

For 1 metre length of the pipe

$$Q_{rad} = q_{rad}A = 462 \times 2 \times \pi \times 0.01 = 29.1\ W/m^2$$

A value of $h = 6\ W/m^2\ K$ is representative of free convection from a tube of this diameter. The heat loss by (black-body) radiation is seen to be comparable to that by convection.

P.S. Since the questions are from different books/sources, symbols/ denotations may be different from the ones that you are familiar. Solve them by yourself. Answers are given just for checking.